NEGATIVE EXPONENTS AND SCIENTIFIC NOTATION

When the exponent in the denominator is larger than the exponent of the numerator, applying the quotient rule will result in a negative exponent. For example,

$$\frac{x^5}{x^7} = x^{5-7} = x^{-2}$$

But we could also evaluate it as follows.

$$\frac{x^5}{x^7} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x} = \frac{1}{x^2}$$

From this we see that

$$x^{-2} = \frac{1}{x^2}$$

In general, we then define negative exponents to be the reciprocal of the base raised to the positive exponent.

Negative Exponents

If *a* is a real number other than 0 and *m* and *n* are positive integers, then

$$a^{-n} = \frac{1}{a^n} \quad \text{or} \quad \frac{1}{a^{-m}} = a^m$$

Example: Evaluating Expressions with Negative exponents

Evaluate:

a. 3^{-2} b. $2x^{-3}$ c. $(3a)^{-1}$

Solution:

a. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ b. $2x^{-3} = \frac{2}{x^3}$ c. $(3a)^{-1} = \frac{1}{3a}$

SCIENTIFIC NOTATION

Very large and very small numbers occur frequently in nature. For example the distance from Earth to the Sun is very large, whereas the diameter of a helium atom is very small. Some calculators do not operate with very large or very small numbers, so **scientific notation** is a shorthand way of writing these numbers so all calculators can handle them.

Scientific Notation

A positive number is written in scientific notation if it is written as the product of a number *a*, where $1 \le a < 10$ and an integer power of *b* of 10:

$$a \times 10^b$$

The following are examples of numbers written in scientific notation.

2.3×10^4

1.6×10^{-3}

To evaluate 2.3×10^4 means that we move the decimal place 4 places to the right.
Thus, $2.3 \times 10^4 = 23{,}000$.

To evaluate 1.6×10^{-3} means that we move the decimal 3 places to the left.
Therefore, $1.6 \times 10^{-3} = 0.0016$

Writing a Number in Standard Notation from Scientific Notation

Move the decimal point in the number the same number of places as the exponent on 10.
If the exponent is positive, move the decimal point to the right.
If the exponent is negative, move the decimal point to the left.

Example: Changing to Standard Notation
Write each number in standard notation
a. 5.24×10^6 b. 1.98×10^{-3}

Solution:
a. $5.24 \times 10^6 = 5{,}240{,}000$
Notice the decimal point has been moved 6 places to the right.

b. $1.98 \times 10^{-3} = 0.00198$
Notice that the decimal has been moved 3 places to the left.

Similarly to write a number in scientific notation we reverse the process.

Writing a Number in Scientific Notation

1. **Move the decimal point in the original number until the new number has a value between 1 and 10.**
2. **Count the number of decimal places the decimal point was moved in Step 1. If the decimal point was moved to the left, the count is positive. If the decimal point was moved to the right, the count is negative.**
3. **Write the product of the new number in Step 1 and 10 raised to an exponent equal to the count found in Step 2.**

* Note: If the original number is "small" then the exponent of 10 will be negative. If the original number is "big" then the exponent of 10 will be positive.

Example: Changing a number to scientific notation

Write each number in scientific notation

a. $63{,}000{,}000$ b. 0.000017

Solution:

a. $63{,}000{,}000 = 6.3 \times 10^{7}$ b. $0.000017 = 1.7 \times 10^{-5}$